Introduction

- As we have seen in the previous lectures writing concurrent programs can lead to violation of progress and safety properties.
- It is important we write correct programs and also verify that they do not have progress and safety violations.
- To achieve this we first build and verify models and then implement the verified models in a language such as Java – this is a common practice in engineering.
- Because it is very difficult to test concurrent programs due to their non-deterministic behaviour, verification of the model becomes even more significant.
- Design → Model → Verify → Implement → Test.
models

A model is a simplified representation of the real world.

Engineers use models to gain confidence in the adequacy and validity of a proposed design.

♦ focus on an aspect of interest - concurrency
♦ model animation to visualise a behaviour
♦ mechanical verification of properties (safety & progress)

Models are described using state machines, known as Labelled Transition Systems \textit{LTS}. These are described textually as finite state processes (\textit{FSP}) and displayed and analysed by the \textit{LTSA} analysis tool.

Modeling Processes

Models are described using state machines, known as Labelled Transition Systems \textit{LTS}. These are described textually as finite state processes (\textit{FSP}) and displayed and analysed by the \textit{LTSA} analysis tool.

♦ \textit{LTS} - graphical form
♦ \textit{FSP} - algebraic form

In FSP a process has \textit{actions} and \textit{states}, on executing an action a process moves from one state to another. In FSP actions are written in lower case and states in upper case.
FSP - action prefix

If $x$ is an action and $P$ a process then $(x \rightarrow P)$ describes a process that initially engages in the action $x$ and then behaves exactly as described by $P$.

**ONESHOT** = (once $\rightarrow$ STOP).

**ONESHOT state machine**

(terminating process)

Convention: actions begin with lowercase letters

PROCESSES begin with uppercase letters

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FSP - action prefix & recursion

Repetitive behaviour uses recursion:

\[
\begin{align*}
\text{SWITCH} & = \text{OFF}, \\
\text{OFF} & = (\text{on} \rightarrow \text{ON}), \\
\text{ON} & = (\text{off} \rightarrow \text{OFF}).
\end{align*}
\]

Substituting to get a more succinct definition:

\[
\begin{align*}
\text{SWITCH} & = \text{OFF}, \\
\text{OFF} & = (\text{on} \rightarrow (\text{off} \rightarrow \text{OFF})).
\end{align*}
\]

And again:

\[
\text{SWITCH} = (\text{on} \rightarrow \text{off} \rightarrow \text{SWITCH}).
\]
animation using LTSA

The $LTS\! A$ animator can be used to produce a trace. Ticked actions are eligible for selection.

In the LTS, the last action is highlighted in red.

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FSP - action prefix

FSP model of a traffic light:

$$TRAFFICLIGHT = (\text{red} \rightarrow \text{orange} \rightarrow \text{green} \rightarrow \text{orange} \rightarrow \text{TRAFFICLIGHT}).$$

LTS generated using $LTS\! A$:

Trace:

$$\text{red} \rightarrow \text{orange} \rightarrow \text{green} \rightarrow \text{orange} \rightarrow \text{red} \rightarrow \text{orange} \rightarrow \text{green} \rightarrow \cdots.$$
If \( x \) and \( y \) are actions then \((x \rightarrow P \mid y \rightarrow Q)\) describes a process which initially engages in either of the actions \( x \) or \( y \). After the first action has occurred, the subsequent behavior is described by \( P \) if the first action was \( x \) and \( Q \) if the first action was \( y \).

Who or what makes the choice?

FSP model of a drinks machine:

\[
\text{DRINKS} = (\text{red} \rightarrow \text{coffee} \rightarrow \text{DRINKS} \mid \text{blue} \rightarrow \text{tea} \rightarrow \text{DRINKS})
\]

LTS generated using \( LTSA \):

Possible traces?
Non-deterministic choice

Process \((x \rightarrow P \mid x \rightarrow Q)\) describes a process which engages in \(x\) and then behaves as either \(P\) or \(Q\).

\[
\text{COIN} = (\text{toss} \rightarrow \text{HEADS} \mid \text{toss} \rightarrow \text{TAILS}), \\
\text{HEADS} = (\text{heads} \rightarrow \text{COIN}), \\
\text{TAILS} = (\text{tails} \rightarrow \text{COIN}).
\]

Tossing a coin.

Possible traces?

FSP - indexed processes and actions

Single slot buffer that inputs a value in the range 0 to 3 and then outputs that value:

\[
\text{BUFF} = (\text{in}[i:0..3] \rightarrow \text{out}[i] \rightarrow \text{BUFF}).
\]
equivalent to

\[
\text{BUFF} = (\text{in}[0] \rightarrow \text{out}[0] \rightarrow \text{BUFF} \\
|\text{in}[1] \rightarrow \text{out}[1] \rightarrow \text{BUFF} \\
|\text{in}[2] \rightarrow \text{out}[2] \rightarrow \text{BUFF} \\
|\text{in}[3] \rightarrow \text{out}[3] \rightarrow \text{BUFF})
\]

or using a process parameter with default value:

\[
\text{BUFF}(N=3) = (\text{in}[i:0..N] \rightarrow \text{out}[i] \rightarrow \text{BUFF}).
\]
The choice \((\text{when } B \ x \rightarrow P \ | \ y \rightarrow Q)\) means that when the guard \(B\) is true then the actions \(x\) and \(y\) are both eligible to be chosen, otherwise if \(B\) is false then the action \(x\) cannot be chosen.

**FSP - guarded actions**

**COUNT \((N=3)\)**

\[
\begin{align*}
\text{COUNT} \ &= \ \text{COUNT}[0], \\
\text{COUNT}[i:0..N] &= (\text{when } (i<N) \ \text{inc} \rightarrow \text{COUNT}[i+1] \\
&\quad | \text{when } (i>0) \ \text{dec} \rightarrow \text{COUNT}[i-1])
\end{align*}
\]

**Guarded actions used to model**

- **Condition**
- **Synchronization**

How to implement in Java?

A countdown timer which beeps after \(N\) ticks, or can be stopped.

**COUNTDOWN \((N=3)\)**

\[
\begin{align*}
\text{COUNTDOWN} \ &= \ (\text{start} \rightarrow \text{COUNTDOWN}[N]), \\
\text{COUNTDOWN}[i:0..N] &= \\
&\quad (\text{when } (i>0) \ \text{tick} \rightarrow \text{COUNTDOWN}[i-1] \\
&\quad | \text{when } (i==0) \ \text{beep} \rightarrow \text{STOP} \\
&\quad | \text{stop} \rightarrow \text{STOP})
\end{align*}
\]
The alphabet of a process is the set of actions in which it can engage.

Alphabet extension can be used to extend the **implicit** alphabet of a process:

\[
\text{WRITER} = (\text{write}[1] \rightarrow \text{write}[3] \rightarrow \text{WRITER}) + \{\text{write}[0..3]\}.
\]

Alphabet of \text{WRITER} is the set \{\text{write}[0..3]\} (we make use of alphabet extensions in later chapters)

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**Implementing the Model**

Modeling **processes** as finite state machines using FSP/LTS.

Implementing **threads** in Java.

**Note:** to avoid confusion, we use the term **process** when referring to the models, and **thread** when referring to the implementation in Java.
Modelling concurrent execution using parallel composition - action interleaving

If P and Q are processes then \((P \parallel Q)\) represents the concurrent execution of P and Q. The operator \(\parallel\) is the parallel composition operator.

\[
\begin{align*}
\text{READ} & = (\text{read} \rightarrow \text{STOP}). \\
\text{CONVERSE} & = (\text{think} \rightarrow \text{talk} \rightarrow \text{STOP}). \\
\text{||CONVERSE_READ} & = (\text{READ} \parallel \text{CONVERSE}).
\end{align*}
\]

Possible traces as a result of action interleaving.

LTS? think \(\rightarrow\) talk \(\rightarrow\) read

\[
\text{think} \rightarrow \text{read} \rightarrow \text{talk}
\]

\[
\text{read} \rightarrow \text{think} \rightarrow \text{talk}
\]

parallel composition - algebraic laws

\[
\begin{align*}
\text{Commutative:} & \quad (P \parallel Q) = (Q \parallel P) \\
\text{Associative:} & \quad (P \parallel (Q \parallel R)) = ((P \parallel Q) \parallel R) \\
& \quad = (P \parallel Q \parallel R).
\end{align*}
\]

Clock radio example:

\[
\begin{align*}
\text{CLOCK} & = (\text{tick} \rightarrow \text{CLOCK}). \\
\text{RADIO} & = (\text{on} \rightarrow \text{off} \rightarrow \text{RADIO}). \\
\text{||CLOCK_RADIO} & = (\text{CLOCK} \parallel \text{RADIO}).
\end{align*}
\]

LTS? Traces? Number of states?
modeling interaction - shared actions

If processes in a composition have actions in common, these actions are said to be *shared*. Shared actions are the way that process interaction is modeled. While unshared actions may be arbitrarily interleaved, a shared action must be executed at the same time by all processes that participate in the shared action.

\[
\text{MAKER} = (\text{make} \rightarrow \text{ready} \rightarrow \text{MAKER}) . \\
\text{USER} = (\text{ready} \rightarrow \text{use} \rightarrow \text{USER}) . \\
\text{MAKER} \quad \text{synchronizes with USER when ready.}
\]

\[
\| \text{MAKER} \_ \text{USER} = (\text{MAKER} \| \| \text{USER}) .
\]

**LTS? Traces? Number of states?**

\[
\text{MAKER}v2 = (\text{make} \rightarrow \text{ready} \rightarrow \text{used} \rightarrow \text{MAKER}v2) . \\
\text{USER}v2 = (\text{ready} \rightarrow \text{use} \rightarrow \text{used} \rightarrow \text{USER}v2) . \\
\| \| \text{MAKER} \_ \text{USER}v2 = (\text{MAKER}v2 \| \| \text{USER}v2) .
\]

**modeling interaction - handshake**

A handshake is an action acknowledged by another:

\[
\text{MAKER}v2 = (\text{make} \rightarrow \text{ready} \rightarrow \text{used} \rightarrow \text{MAKER}v2) . \quad 3 \text{ states} \\
\text{USER}v2 = (\text{ready} \rightarrow \text{use} \rightarrow \text{used} \rightarrow \text{USER}v2) . \quad 3 \text{ states}
\]

\[
\| \| \text{MAKER} \_ \text{USER}v2 = (\text{MAKER}v2 \| \| \text{USER}v2) . \quad 3 \times 3 \text{ states?}
\]

**Diagram**

- State 0: make
- State 1: ready
- State 2: use
- State 3: used

Interaction constrains the overall behaviour.
modeling interaction - multiple processes

Multi-party synchronization:

\[
\begin{align*}
\text{MAKE}_A & \ = \ (\text{makeA} \rightarrow \text{ready} \rightarrow \text{used} \rightarrow \text{MAKE}_A). \\
\text{MAKE}_B & \ = \ (\text{makeB} \rightarrow \text{ready} \rightarrow \text{used} \rightarrow \text{MAKE}_B). \\
\text{ASSEMBLE} & \ = \ (\text{ready} \rightarrow \text{assemble} \rightarrow \text{used} \rightarrow \text{ASSEMBLE}).
\end{align*}
\]

\[\text{FACTORY} = (\text{MAKE}_A \ || \ \text{MAKE}_B \ || \ \text{ASSEMBLE}).\]

composite processes

A composite process is a parallel composition of primitive processes. These composite processes can be used in the definition of further compositions.

\[
\begin{align*}
\text{FACTORY} & \ = \ (\text{MAKE}_A \ || \ \text{MAKE}_B). \\
\text{FACTORY} & \ = \ (\text{MAKERS} \ || \ \text{ASSEMBLE}).
\end{align*}
\]

Substituting the definition for \text{MAKERS} in \text{FACTORY} and applying the commutative and associative laws for parallel composition results in the original definition for \text{FACTORY} in terms of primitive processes.

\[
\text{FACTORY} = (\text{MAKE}_A \ || \ \text{MAKE}_B \ || \ \text{ASSEMBLE}).
\]

\[\]
process labeling

alpha:P prefixes each action label in the alphabet of P with a.

Two instances of a switch process:

\[ \text{SWITCH} = (\text{on} \rightarrow \text{off} \rightarrow \text{SWITCH}). \]
\[ \text{||TWO\_SWITCH} = (a: \text{SWITCH} \parallel b: \text{SWITCH}). \]

An array of instances of the switch process:

\[ \text{||SWITCHES(N=3)} = (\forall i:1..N \ s[i]: \text{SWITCH}). \]
\[ \text{||SWITCHES(N=3)} = (s[i:1..N]: \text{SWITCH}). \]

process labeling by a set of prefix labels

\{a1,..,ax\}:P replaces every action label n in the alphabet of P with the labels a1.n,..,ax.n. Further, every transition (n->X) in the definition of P is replaced with the transitions ({a1.n,..,ax.n} ->X).

Process prefixing is useful for modeling shared resources:

\[ \text{RESOURCE} = (\text{acquire} \rightarrow \text{release} \rightarrow \text{RESOURCE}). \]
\[ \text{USER} = (\text{acquire} \rightarrow \text{use} \rightarrow \text{release} \rightarrow \text{USER}). \]
\[ \text{||RESOURCE\_SHARE} = (a: \text{USER} \parallel b: \text{USER} \parallel \{a, b\}: : \text{RESOURCE}). \]
process prefix labels for shared resources

How does the model ensure that the user that acquires the resource is the one to release it?

action relabeling

Relabeling functions are applied to processes to change the names of action labels. The general form of the relabeling function is:

\[\{\text{newlabel}_1/\text{oldlabel}_1, \ldots, \text{newlabel}_n/\text{oldlabel}_n\}\].

Relabeling to ensure that composed processes synchronize on particular actions.

CLIENT = (call→wait→continue→CLIENT).
SERVER = (request→service→reply→SERVER).
action relabeling

\[ ||\text{CLIENT\_SERVER} = (\text{CLIENT} \mid \mid \text{SERVER}) \]

/\{call/request, reply/wait\}.

An alternative formulation of the client server system is described below using qualified or prefixed labels:

\[
\text{SERVERv2} = (\text{accept.request} \rightarrow \text{service} \rightarrow \\
\hspace{1cm} \text{accept.reply} \rightarrow \text{SERVERv2}).
\]

\[
\text{CLIENTv2} = (\text{call.request} \rightarrow \text{call.reply} \rightarrow \text{continue} \rightarrow \text{CLIENTv2}).
\]

\[ ||\text{CLIENT\_SERVERv2} = (\text{CLIENTv2} \mid \mid \text{SERVERv2}) \]

/\{call/accept\}. 
**action hiding** - abstraction to reduce complexity

When applied to a process $P$, the hiding operator $\{a_1..a_x\}$ removes the action names $a_1..a_x$ from the alphabet of $P$ and makes these concealed actions "silent". These silent actions are labeled $\tau$. Silent actions in different processes are not shared.

Sometimes it is more convenient to specify the set of labels to be *exposed*...

When applied to a process $P$, the interface operator $@\{a_1..a_x\}$ hides all actions in the alphabet of $P$ not labeled in the set $a_1..a_x$.

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**action hiding**

The following definitions are equivalent:

$$\text{USER} = (\text{acquire} \rightarrow \text{use} \rightarrow \text{release} \rightarrow \text{USER}) \setminus \{\text{use}\}.$$  

$$\text{USER} = (\text{acquire} \rightarrow \text{use} \rightarrow \text{release} \rightarrow \text{USER}) \ @ \{\text{acquire}, \text{release}\}.$$  

Minimization removes hidden $\tau$ actions to produce an LTS with equivalent observable behavior.
FSP for verification – an example

Consider the following FSP which shows two process P and Q using two shared resources – a printer and a scanner

RESOURCE = (get-> put-> RESOURCE).

Q = (scanner.get -> printer.get -> copy-> scanner.put -> printer.put -> Q).

||SYS = (p:P||q:Q ||{p,q}::printer:RESOURCE ||{p,q}::scanner:RESOURCE).

How to verify whether this will work correctly? We run the model using LTSA. LTSA tool will allow us to check the safety (deadlocks?) and progress of this model. This can even show you the trace to deadlock.

FSP for verification – example 2

Consider the following FSP for a bounded buffer which is shared by a Producer and a Consumer

//FSP for bounded buffer
const EMPTY = 0
cost MAX_SIZE = 10

BUFF = BUFFER[EMPTY],
BUFFER[size:EMPTY..MAX_SIZE] = (when (size<MAX_SIZE) put -> BUFFER[size + 1]
|when (size > EMPTY) get -> BUFFER[size - 1] )

PRODUCER = (put -> PRODUCER).
CONSUMER = (get -> CONSUMER).

||PRODUCER_CONSUMER = (PRODUCER || CONSUMER || BUFF).

How to verify whether this will work correctly? We run the model using LTSA. LTSA tool will allow us to check the safety (deadlocks?) and progress of this model. This can even show you the trace to deadlock or other problems such as progress violations.
Resources and references

- Concurrency: State Models & Java Programs, 2e, Jeff Magee & Jeff Kramer, Wiley
- Course Website – http://sfdv3006.wikidot.com